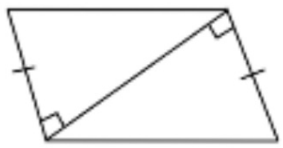
Do Now

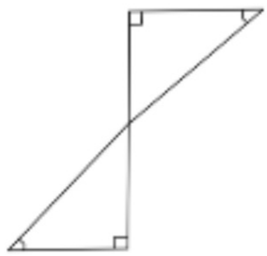
Calculators

State if the triangles are congruent. If they are, state how you know.

a)

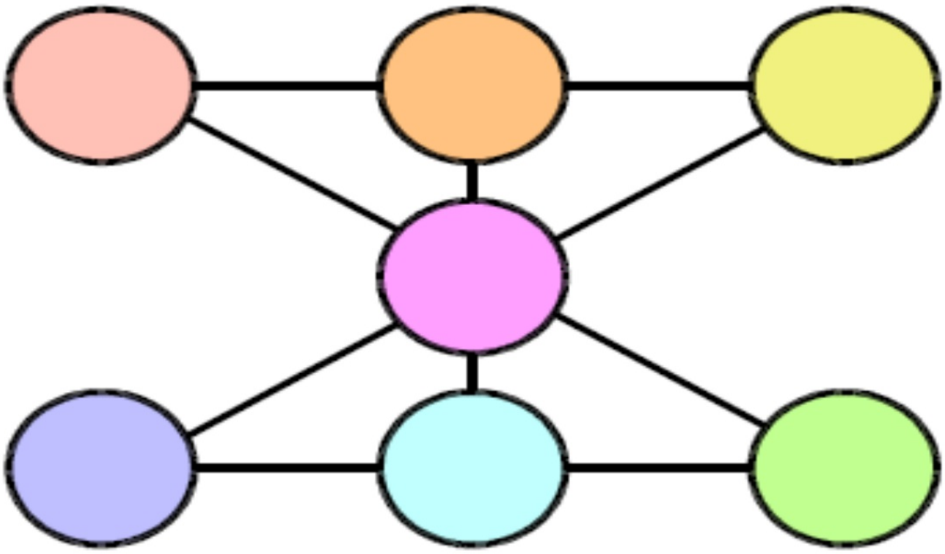


b)



Check In!

Can you put the numbers 1 to 7 in each circle so that the total of every line is 12?

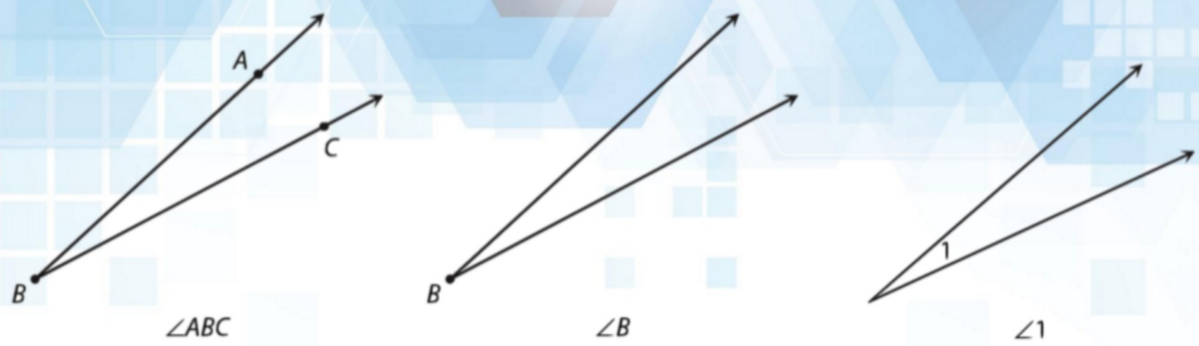


Introduction

Think about crossing a pair of chopsticks and the angles that are created when they are opened at various positions. How many angles are formed? What are the relationships among those angles? This lesson explores angle relationships. We will be examining the relationships of angles that lie in the same plane. A **plane** is a two-dimensional figure, meaning it is a flat surface, and it extends infinitely in all directions. Planes require at least three non-collinear points. Planes are named using those points or a capital script letter. Since they are flat, planes have no depth.

Key Concepts

- Angles can be labeled with one point at the vertex, three points with the vertex point in the middle, or with numbers. See the examples that follow.

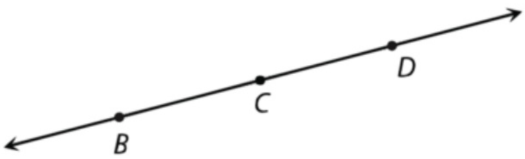
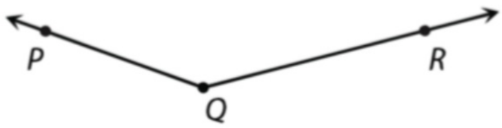


Key Concepts, *continued*

- Be careful when using one vertex point to name the angle, as this can lead to confusion.
- If the vertex point serves as the vertex for more than one angle, three points or a number must be used to name the angle.

Key Concepts, *continued*

- **Straight angles** are angles with rays in opposite directions—in other words, straight angles are straight lines.

<table>
<tr><th>Straight angle</th><th>Not a straight angle</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>∠BCD is a straight angle. Points B, C, and D lie on the same line.</td><td>∠PQR is not a straight angle. Points P, Q, and R do not lie on the same line.</td></tr>
</table>

Key Concepts, *continued*

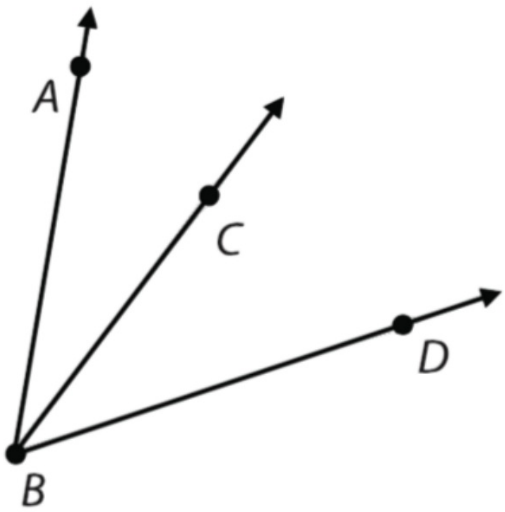
- **Adjacent angles** are angles that lie in the same plane and share a vertex and a common side. They have no common interior points.
- **Nonadjacent** angles have no common vertex or common side, or have shared interior points.

Key Concepts, *continued*

Adjacent angles

$\angle ABC$ is adjacent to $\angle CBD$. They share vertex B and $\overrightarrow{BC}$.

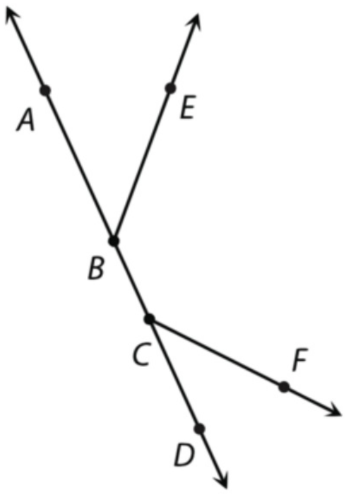
$\angle ABC$ and $\angle CBD$ have no common interior points.



Key Concepts, *continued*

Nonadjacent angles

∠*ABE* is not adjacent to ∠*FCD*.
They do not have a common vertex.



(*continued*)

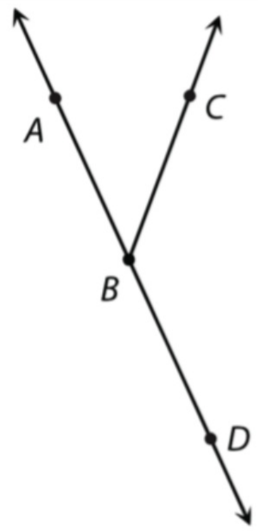
Key Concepts, *continued*

- **Linear pairs** are pairs of adjacent angles whose non-shared sides form a straight angle.

Key Concepts, *continued*

Linear pair

$\angle ABC$ and $\angle CBD$ are a linear pair. They are adjacent angles with non-shared sides, creating a straight angle.



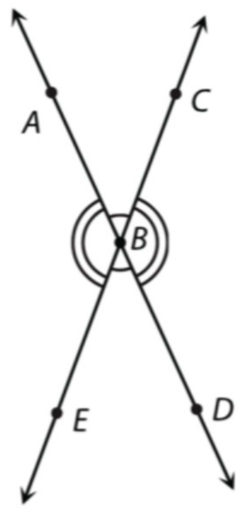
Key Concepts, *continued*

- **Vertical angles** are nonadjacent angles formed by two pairs of opposite rays.

Theorem
Vertical Angles Theorem Vertical angles are congruent.

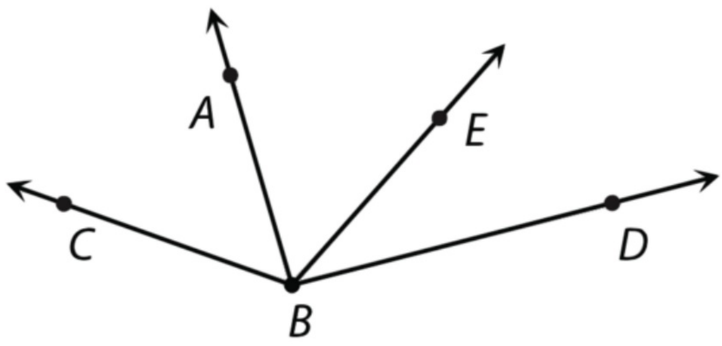
Key Concepts, *continued*

Vertical angles
$\angle ABC$ and $\angle EBD$ are vertical angles. $\angle ABC \cong \angle EBD$ $\angle ABE$ and $\angle CBD$ are vertical angles. $\angle ABE \cong \angle CBD$



Key Concepts, *continued*

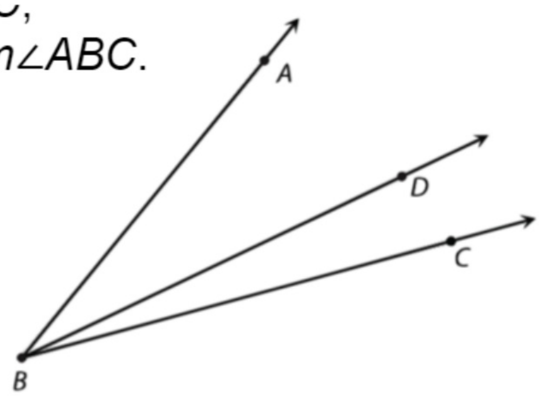
Not vertical angles



$\angle ABC$ and $\angle EBD$ are not vertical angles. $\overrightarrow{BC}$ and $\overrightarrow{BD}$ are not opposite rays. They do not form one straight line.

Key Concepts, *continued*

Postulate
Angle Addition Postulate If D is in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$. If $m\angle ABD + m\angle DBC = m\angle ABC$, then D is in the interior of $\angle ABC$.



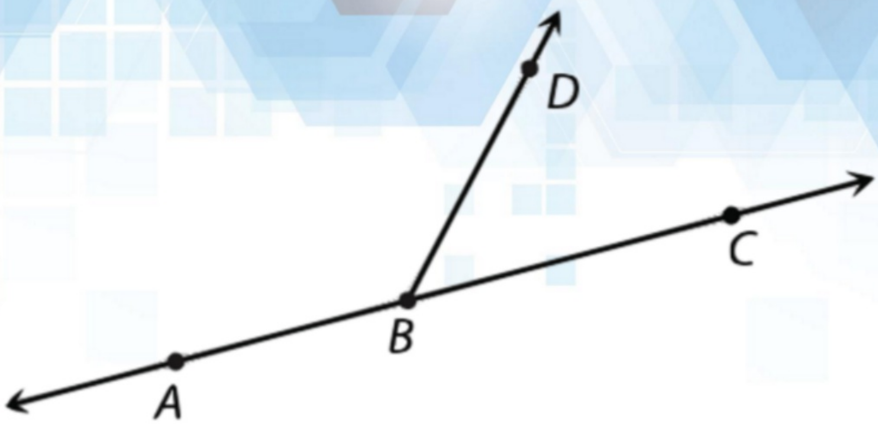
Key Concepts, *continued*

- Informally, the Angle Addition Postulate means that the measure of the larger angle is made up of the sum of the two smaller angles inside it. **Postulates** are true statements that don't need proofs.
- **Supplementary angles** are two angles whose sum is 180°.
- Supplementary angles can form a linear pair or be nonadjacent.

Key Concepts, *continued*

- In the diagram below, the angles form a linear pair.

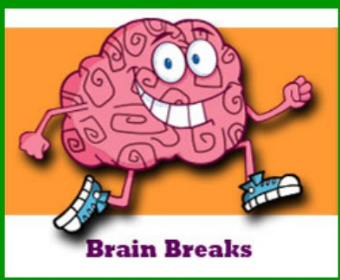
$m\angle ABD + m\angle DBC = 180$



Key Concepts, *continued*

Theorem
Supplement Theorem If two angles form a linear pair, then they are supplementary.

Brain Break!



Key Concepts, *continued*

- Angles have the same congruence properties that segments do.

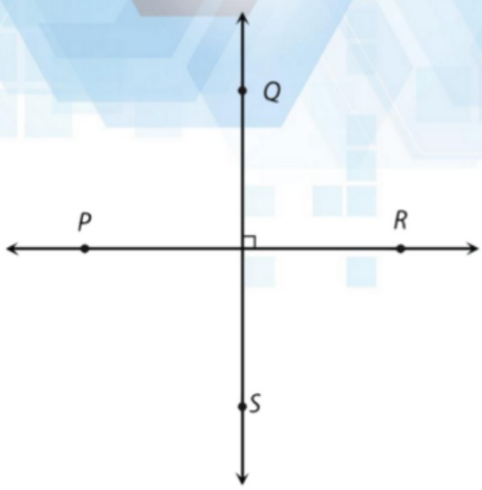
Theorem
Congruence of angles is reflexive, symmetric, and transitive. • Reflexive Property: $\angle 1 \cong \angle 1$ • Symmetric Property: If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$. • Transitive Property: If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

Key Concepts, *continued*

Theorem
Angles supplementary to the same angle or to congruent angles are congruent. If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.

Key Concepts, *continued*

- The symbol for indicating perpendicular lines in a diagram is a box at one of the right angles, as shown below.



Key Concepts, *continued*

- The symbol for writing perpendicular lines is ⊥, and is read as "is perpendicular to."
- In the diagram, $\overleftrightarrow{SQ} \perp \overleftrightarrow{PR}$.
- Rays and segments can also be perpendicular.
- In a pair of perpendicular lines, rays, or segments, only one right angle box is needed to indicate perpendicular lines.

Key Concepts, *continued*

- **Perpendicular bisectors** are lines that intersect a segment at its midpoint at a right angle; they are perpendicular to the segment.
- Any point along the perpendicular bisector is **equidistant**, or the same distance, from the endpoints of the segment that it bisects.

Key Concepts, *continued*

Theorem
Perpendicular Bisector Theorem If a point lies on the perpendicular bisector of a segment, then that point is equidistant from the endpoints of the segment. If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.

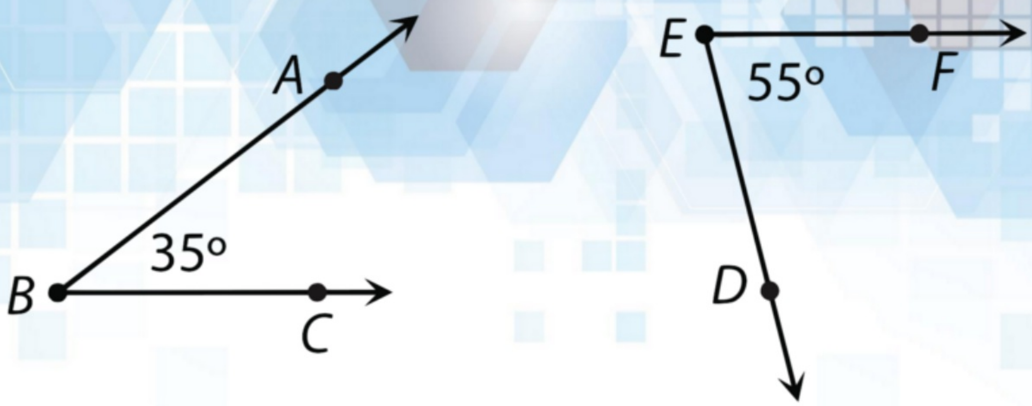
(*continued*)

Key Concepts, *continued*

- **Complementary angles** are two angles whose sum is 90º.
- Complementary angles can form a right angle or be nonadjacent.
- The following diagram shows a pair of nonadjacent complementary angles.

Key Concepts, *continued*

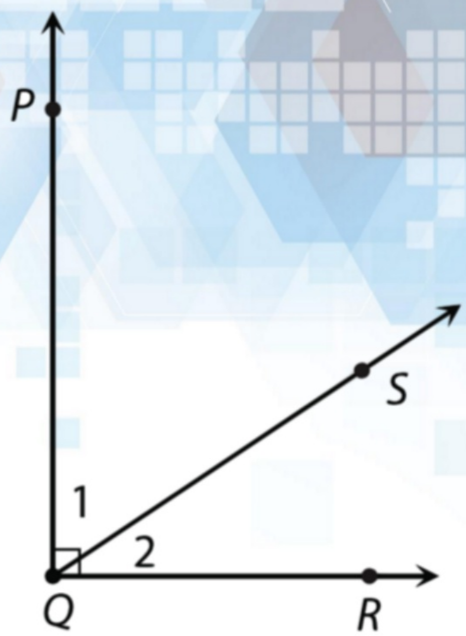
$$m\angle B + m\angle E = 90$$



Key Concepts, *continued*

- The diagram at right shows a pair of adjacent complementary angles labeled with numbers.

$$m\angle 1 + m\angle 2 = 90$$



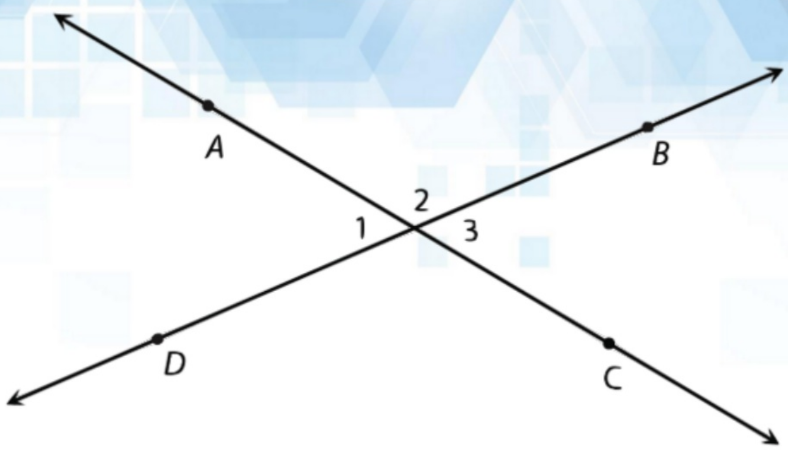
Guided Practice

Example 4

Prove that vertical angles are congruent given a pair of intersecting lines, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$.

Guided Practice: Example 4, *continued*

1. Draw a diagram and label three adjacent angles.



Guided Practice: Example 4, *continued*

2. Start with the Supplement Theorem.

Supplementary angles add up to 180º.

$$m\angle 1 + m\angle 2 = 180$$

$$m\angle 2 + m\angle 3 = 180$$

Guided Practice: Example 4, *continued*

3. Use substitution.

Both expressions are equal to 180, so they are equal to each other. Rewrite the first equation, substituting $m\angle 2 + m\angle 3$ in for 180.

$$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$$

Guided Practice: Example 4, *continued*

4. Use the Reflexive Property.

$m\angle 2 = m\angle 2$

Guided Practice: Example 4, *continued*

5. Use the Subtraction Property.

Since $m\angle 2 = m\angle 2$, these measures can be subtracted out of the equation $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$.

This leaves $m\angle 1 = m\angle 3$.

Guided Practice: Example 4, *continued*

6. Use the definition of congruence.

Since $m\angle 1 = m\angle 3$, by the definition of congruence, $\angle 1 \cong \angle 3$.

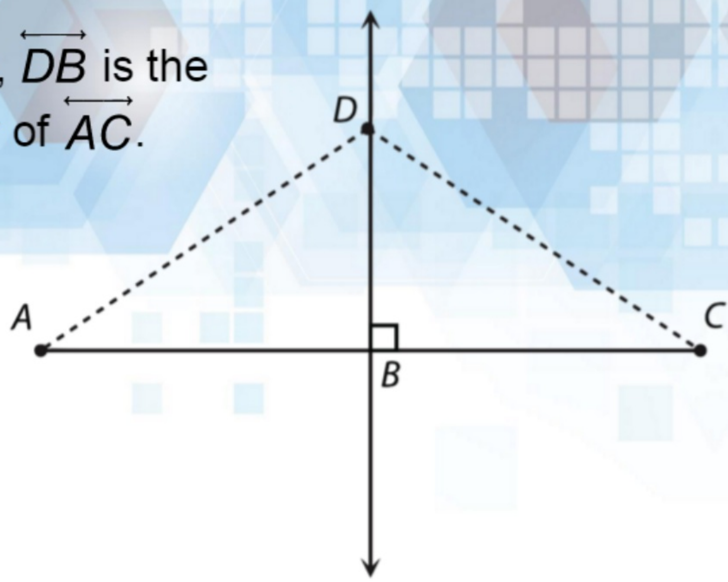
$\angle 1$ and $\angle 3$ are vertical angles and they are congruent. This proof also shows that angles supplementary to the same angle are congruent.

Guided Practice

Example 5

In the diagram at right, $\overleftrightarrow{DB}$ is the perpendicular bisector of $\overleftrightarrow{AC}$.

If $AD = 4x - 1$ and $DC = x + 11$, what are the values of AD and DC?



Guided Practice: Example 5, *continued*

1. **Use the Perpendicular Bisector Theorem to determine the values of *AD* and *DC*.**

   If a point is on the perpendicular bisector of a segment, then that point is equidistant from the endpoints of the segment being bisected. That means $AD = DC$.

Guided Practice: Example 5, *continued*

2. Use substitution to solve for *x*.

$AD = 4x - 1$ and $DC = x + 11$	Given equations
$AD = DC$	Perpendicular Bisector Theorem
$4x - 1 = x + 11$	Substitute $4x - 1$ for AD and $x + 11$ for DC.
$3x = 12$	Combine like terms.
$x = 4$	Divide both sides of the equation by 3.

Guided Practice: Example 5, *continued*

3. **Substitute the value of *x* into the given equations to determine the values of *AD* and *DC*.**

$AD = 4x - 1$	$DC = x + 11$
$AD = 4(4) - 1$	$DC = (4) + 11$
$AD = 15$	$DC = 15$

AD and *DC* are each 15 units long.