Introduction

You may recall that a line is the graph of a linear function and that all linear functions can be written in the form $f(x) = mx + b$, in which m is the slope and b is the y-intercept. The solutions to a linear function are the infinite set of points on the line. In this lesson, you will learn about a second type of function known as a quadratic function.

Key Concepts

- A **quadratic function** is a function that can be written in the form $f(x) = ax^2 + bx + c$, where x is the variable, a, b, and c are constants, and $a \neq 0$. This form is also known as the **standard form of a quadratic function**, where a is the coefficient of the quadratic term, b is the coefficient of the linear term, and c is the constant term.
- Quadratic functions can be graphed on a coordinate plane.

Key Concepts, *continued*

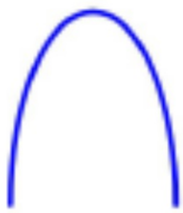
- One method of graphing a quadratic function is to create a table of at least five *x*-values and calculate the corresponding *y*-values.
- Once graphed, all quadratic functions will have a U-shape called a **parabola**.
- Distinguishing characteristics can be used to describe, draw, and compare quadratic functions. These characteristics include the *y*-intercept, *x*-intercepts, the maximum or minimum of the function, and the axis of symmetry.

Quadratic Equations

Standard Form: $ax^2 + bx + c$; where $a = 1$

Shape:

a >1



a<1

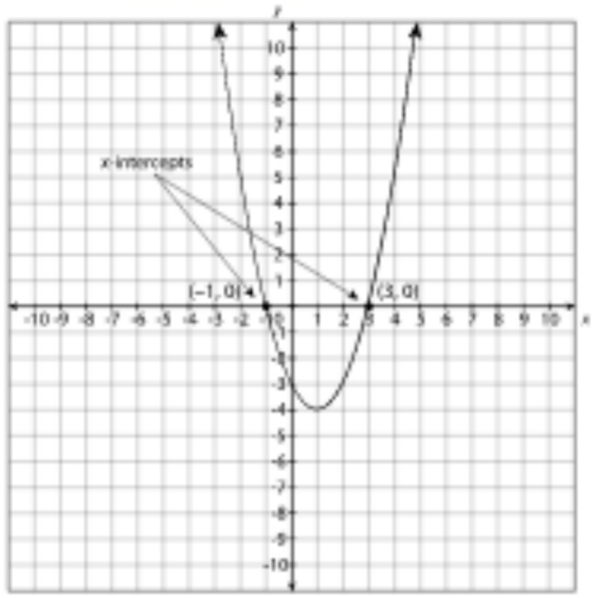
Factor: means to write quadratic into two binomials multiplied together.

Key Concepts, *continued*

- The **intercept** of a graph is the point at which a line intercepts the *x*- or *y*-axis.
- The ***x*-intercept** is the point at which a graph crosses the *x*-axis. It is written as $(x, 0)$.
- The *x*-intercepts of a quadratic function occur when the parabola intersects the *x*-axis at $(x, 0)$.

Key Concepts, *continued*

- The graph at right of a quadratic function, $f(x) = x^2 - 2x - 3$, shows the location of the parabola's *x*-intercepts.



Key Concepts, *continued*

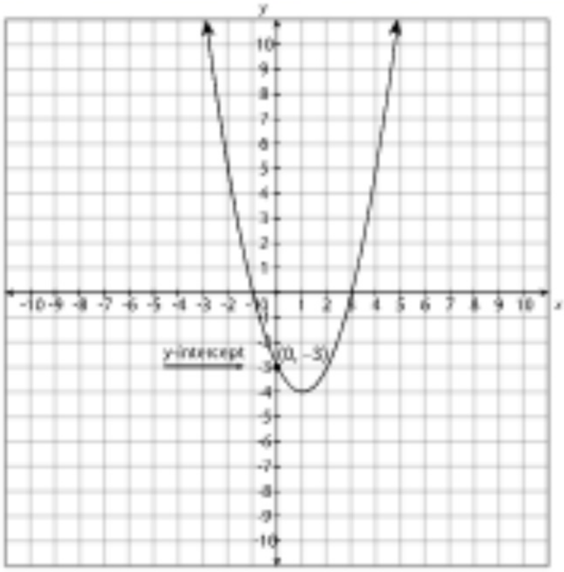
- Note that the x-intercepts of this function are (–1, 0) and (3, 0).
- The equation of the x-axis is $y = 0$; therefore, the x-intercepts can also be found in a table by identifying which values of x have a corresponding y-value that is 0.

Key Concepts, *continued*

- The ***y*-intercept** of a quadratic function is the point at which the graph intersects the *y*-axis. It is written as (0, *y*).
- The *y*-intercept of a quadratic is the *c* value of the quadratic equation when written in standard form.

Key Concepts, *continued*

- The graph at right of a quadratic function, $f(x) = x^2 - 2x - 3$, shows the location of the parabola's *y*-intercept.

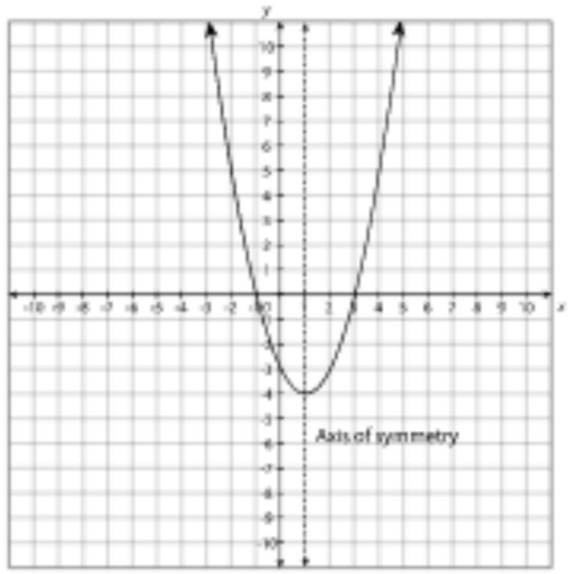


Key Concepts, *continued*

- Note that the *y*-intercept of this equation is (0, –3). The *c* value of the function is also –3.
- The **axis of symmetry of a parabola** is the line through the parabola about which the parabola is symmetric.
- The equation of the axis of symmetry is $x = \frac{-b}{2a}$.

Key Concepts, *continued*

- The equation of the axis of symmetry for the function $f(x) = x^2 - 2x - 3$ is $x = 1$ because the vertical line through 1 is the line that cuts the parabola in half.



Key Concepts, *continued*

- The **vertex of a parabola** is the point on a parabola that is the maximum or minimum of the function.
- The **maximum** is the largest *y*-value of a quadratic equation and the **minimum** is the smallest *y*-value.
- The **extrema** of a graph are the minima or maxima of a function. In other words, an extremum is the function value that achieves either a minimum or maximum.

Key Concepts, *continued*

- The vertex of a quadratic lies on the axis of symmetry.
- The vertex is often written as (h, k).
- The formula $x = \frac{-b}{2a}$ is also used to find the x-coordinate of the vertex.
- To find the y-coordinate, substitute the value of x into the original function, $(h, k) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Guided Practice

Example 2

Given the function $f(x) = -2x^2 + 16x - 30$, identify the key features of the graph: the extremum, vertex, and y-intercept. Then sketch the graph.

Guided Practice: Example 2, *continued*

1. Determine the extremum of the graph.

The extreme value is a minimum when $a > 0$. It is a maximum when $a < 0$.

Because $a = -2$, the graph opens downward and the quadratic has a maximum.

Guided Practice: Example 2, *continued*

2. Determine the vertex of the graph.

The maximum value occurs at the vertex.

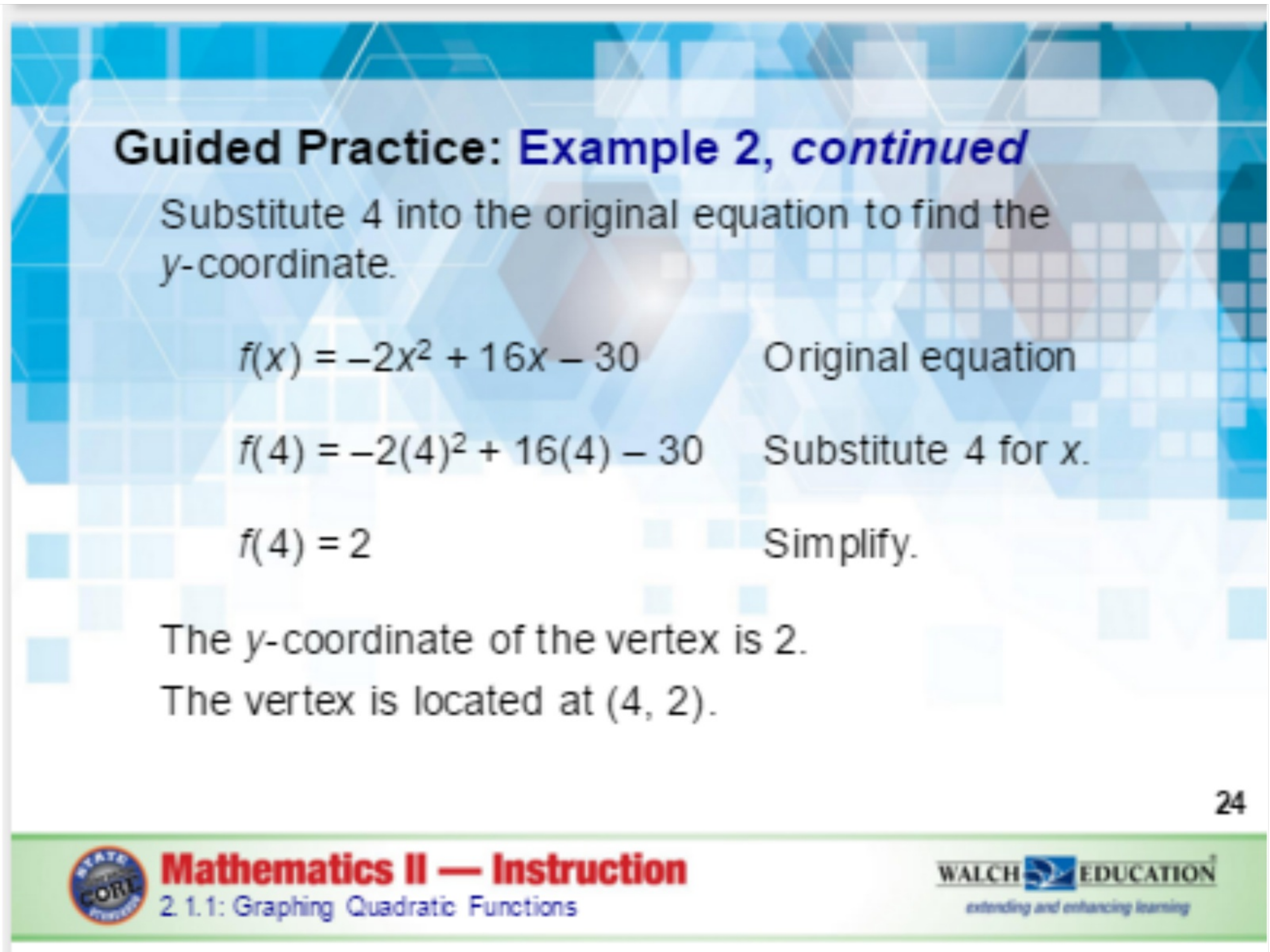
The vertex is of the form $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Use the original equation $f(x) = -2x^2 + 16x - 30$ to find the values of a and b in order to find the x-value of the vertex.

Guided Practice: Example 2, *continued*

$x = \frac{-b}{2a}$	Formula to find the *x*-coordinate of the vertex of a quadratic
$x = \frac{-(16)}{2(-2)}$	Substitute –2 for *a* and 16 for *b*.
$x = 4$	Simplify.

The *x*-coordinate of the vertex is 4.



Guided Practice: Example 2, *continued*
Substitute 4 into the original equation to find the y-coordinate.
$f(x) = -2x^2 + 16x - 30$ Original equation
$f(4) = -2(4)^2 + 16(4) - 30$ Substitute 4 for x.
$f(4) = 2$ Simplify.
The y-coordinate of the vertex is 2.
The vertex is located at (4, 2).
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Guided Practice: Example 2, *continued*

3. Determine the *y*-intercept of the graph.

The *y*-intercept occurs when $x = 0$.

Substitute 0 for *x* in the original equation.

$f(x) = -2x^2 + 16x - 30$	Original equation
$f(0) = -2(0)^2 + 16(0) - 30$	Substitute 0 for *x*.
$f(0) = -30$	Simplify.

The *y*-intercept is (0, –30).

When the quadratic equation is written in standard form, the *y*-intercept is *c*.

Guided Practice

Example 4

Given the function $f(x) = -2x^2 - 12x - 10$, identify the key features of its graph: the extremum, vertex, and y-intercept. Then sketch the graph.