Simplifying Radicals

Rules and Properties: Square Root Expressions in Simplest Form

An expression involving square roots is in *simplest form* if

1. There are no perfect-square factors in a radical.
2. No fraction appears inside a radical.
3. No radical appears in the denominator.

Notes on Simplifying Radicals

Essential Understanding You can simplify a radical expression when the exponent of one factor of the radicand is a multiple of the radical's index.

You can simplify the product of powers that have the same exponent. Similarly, you can simplify the product of radicals that have the same index.

Same Exponent	Same Index
$2^2 \cdot 3^2 = (2 \cdot 3)^2$	$\sqrt{2} \cdot \sqrt{3} = \sqrt{2 \cdot 3}$
$4^3 \cdot 5^3 = (4 \cdot 5)^3$	$\sqrt[3]{4} \cdot \sqrt[3]{5} = \sqrt[3]{4 \cdot 5}$

take note **Property** Combining Radical Expressions: Products

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

Problem 1 Multiplying Radical Expressions

Can you simplify the product of the radical expressions? Explain.

A $\sqrt[3]{6} \cdot \sqrt{2}$

No. The indexes are different. The property above does not apply.

B $\sqrt[3]{-4} \cdot \sqrt[3]{2}$

Yes. $\sqrt[3]{-4} \cdot \sqrt[3]{2} = \sqrt[3]{-4(2)} = \sqrt[3]{-8} = -2$.

If the radicand of $\sqrt[n]{a}$ has a perfect nth power among its factors, you can *reduce* the radical. If you reduce a radical as much as possible, the radical is in **simplest form**. For example, consider $\sqrt{24}$ and $\sqrt[3]{24}$.

$\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{2^2 \cdot 6} = \sqrt{2^2} \cdot \sqrt{6} = 2\sqrt{6}$ $\quad 2\sqrt{6}$ is in simplest form.

$\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{2^3 \cdot 3} = \sqrt[3]{2^3} \cdot \sqrt[3]{3} = 2\sqrt[3]{3}$ $\quad 2\sqrt[3]{3}$ is in simplest form.

Problem 2 Simplifying a Radical Expression

What is the simplest form of $\sqrt[3]{54x^5}$?

$\sqrt[3]{54x^5} = \sqrt[3]{3^3 \cdot 2 \cdot x^2 \cdot x^3}$ Find all perfect cube factors.

$= \sqrt[3]{3^3x^3} \cdot \sqrt[3]{2x^2}$ $\quad \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$= 3x\sqrt[3]{2x^2}$ Simplify.

Got It? 2. What is the simplest form of $\sqrt[3]{128x^7}$?

Simplify $\sqrt{72}$.

Simplify $\sqrt{42}$.

Problem 3 Simplifying a Product

What is the simplest form of $\sqrt{72x^3y^2} \cdot \sqrt{10xy^3}$?

Think

You need to multiply the radicands and find the perfect square factors.

Now find square roots. Since $\sqrt{72x^3y^2}$ and $\sqrt{10xy^3}$ must be real numbers, x and y are nonnegative, so no absolute value symbols are needed.

Write

$\sqrt{72x^3y^2} \cdot \sqrt{10xy^3} = \sqrt{(72x^3y^2)(10xy^3)}$

$= \sqrt{720x^4y^5}$

$= \sqrt{12^2(5)(x^2)^2(y^2)^2y}$

$= \sqrt{12^2(x^2)^2(y^2)^2} \cdot \sqrt{5y}$

$= 12|x^2y^2| \cdot \sqrt{5y}$

$= 12x^2y^2\sqrt{5y}$

The simplest form is $12x^2y^2\sqrt{5y}$.

Simplify.

8. $\sqrt{14}$ 9. $\sqrt{60}$ 10. $\sqrt{54}$

What is the simplest form of $\sqrt{45x^5y^3} \cdot \sqrt{35xy^4}$?

take note

Property Combining Radical Expressions: Quotients

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$.

What is the simplest form of the quotient?

A $\frac{\sqrt{18x^5}}{\sqrt{2x^3}}$

$$\frac{\sqrt{18x^5}}{\sqrt{2x^3}} = \sqrt{\frac{18x^5}{2x^3}}$$
$$= \sqrt{9x^2}$$
$$= 3x$$

What is the simplest form of $\frac{\sqrt{50x^6}}{\sqrt{2x^4}}$?

B $\frac{\sqrt[3]{162y^5}}{\sqrt[3]{3y^2}}$

Another way to simplify a radical expression is to **rationalize the denominator**. You rewrite the expression so that there are no radicals in any denominator and no denominator in any radical.

Multiply by 1.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$$

The product of $\sqrt{2}$ and itself is a rational number, 2.

Problem 5 Rationalizing the Denominator

Multiple Choice What is the simplest form of $\sqrt[3]{\frac{5x^2}{12y^2z}}$?

(A) $\frac{\sqrt[3]{90x^2yz^2}}{6yz}$ (B) $\frac{\sqrt[3]{5x^2}}{\sqrt[3]{12y^2z}}$ (C) $\frac{5\sqrt[3]{x^2yz^2}}{yz}$ (D) $5\sqrt[3]{x^2z}$

What is the simplest form of $\frac{\sqrt[3]{7x}}{\sqrt[3]{5y^2}}$?

Activity

HOMEWORK

Algebra 2

Pg. 370 #10-44 even